Internet Appendix for "Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market"

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Section A of this appendix contains a derivation of the Taylor approximations needed for the proofs of Theorems I to III. Section B describes how we calculate standard errors for the GMM estimates of the model parameters. In Section C we briefly discuss how we use the repeat sales method to construct returns on CDS portfolios. Section D describes the risk-free data used in our analysis and how we construct risk-neutral default probabilities, needed to form CDS returns. Finally, the appendix contains four additional figures.

A. Taylor Approximations

This appendix provides a Taylor approximation to the matrix $V(I)V(\delta_i)^{-1}$ around $\widehat{c}_h = 0$. Empirically, transaction costs are small, in the sense that the standard deviation of c is smaller than the standard deviation of r. Write $\widehat{c}_h = \xi\tilde{c}$ so that

$$V(\delta_i) = V_r - \xi(\delta_i\widetilde{C} + \delta_i\widetilde{C}') + \xi^2 V_{\tilde{c}}, \tag{IA.1}$$

where $V_{\tilde{c}} = Var(\tilde{c})$ and $\widetilde{C} = Cov(\tilde{c}, \widehat{r}_h)$ and use a Taylor approximation around $\xi = 0$:

$$V(I)V(\delta_i)^{-1} \approx V(I)V(\delta_i)^{-1}_{|\xi=0} + \xi\frac{\partial(V(I)V(\delta_i)^{-1})}{\partial\xi}. \tag{IA.2}$$

We have

$$\frac{\partial V(\delta_i)^{-1}}{\partial\xi} = -V(\delta_i)^{-1}\frac{\partial V(\delta_i)}{\partial\xi}V(\delta_i)^{-1} = V(\delta_i)^{-1}(\delta_i\widetilde{C} + \delta_i\widetilde{C}' - 2\xi V_{\tilde{c}})V(\delta_i)^{-1}. \tag{IA.3}$$

Evaluating this in $\xi = 0$ we obtain

$$V(I)V(\delta_i)^{-1} \approx I + \xi(V_r V_r^{-1}(\delta_i\widetilde{C} + \delta_i\widetilde{C}')V_r^{-1}) + \xi(-(\widetilde{C} + \widetilde{C}')V_r^{-1}) \tag{IA.4}$$

$$= I + (\delta_i C + \delta_i C')V_r^{-1} - (C + C')V_r^{-1}.$$

If $\delta_i = -I$ this simplifies to $I - 2H_1$, and if $\delta_i = D$ this equals $I + H_2 - H_1$. Finally, using a similar derivation we obtain that $V(I)V_r^{-1} \approx I - H_1$.

B. GMM Standard Errors

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In this appendix we outline the calculation of standard errors along the lines of Shanken (1992) for the GMM estimation of (16). To avoid complicated notation, we discuss a simplified example; the full derivation is available on request. The simplified example is given by

$$E(r) = X\alpha + E(c)\zeta + \beta\theta, \tag{IA.5}$$

with $\beta = (\beta_{r_h,r_b}\ \beta_{\widehat{r}_h,r_n}\ \beta_{\widehat{c}_h,r_n})$ and $\theta = (\psi, \lambda, \kappa)'$. In practice, the regressors are replaced by estimates and the second-step regression is

$$\overline{er} = X\alpha + \overline{c}\zeta + \widehat{\beta}\theta + \eta, \tag{IA.6}$$

with

$$\eta = (\overline{er} - E(r)) - (\overline{c} - E(c))\zeta - (\widehat{\beta} - \beta)\theta \tag{IA.7}$$

and $\overline{er} = T^{-1}\sum_{t=1}^{T} er_t$, where er_t is our weekly estimate for the expected CDS returns in (22), and likewise for $\overline{c}$.

The second-step estimates for (α, ζ, θ) are given by $(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}\overline{er}$, with $Z = (X\ \overline{c}\ \widehat{\beta})$ and Σ^{-1} the weighting matrix. The standard errors of these estimates are given by $(Z'\Sigma^{-1}Z)^{-1}Z'\Sigma^{-1}Var(\eta)\Sigma^{-1}Z(Z'\Sigma^{-1}Z)^{-1}$, with

$$Var(\eta) = Var(\overline{er}) + \zeta^2 Var(\overline{c}) + Var(\widehat{\beta}\theta). \tag{IA.8}$$

The elements $Var(\overline{er})$ and $Var(\overline{c})$ can be estimated using a Newey-West procedure,

$$Var(\overline{er}) \mathrel{\widehat{=}} T^{-2}\sum_t \sum_k w_k (er_t - \overline{er})(er_{t-k} - \overline{er}), \tag{IA.9}$$

and likewise for $\overline{c}$, and

$$Var(\widehat{\beta}\theta) \mathrel{\widehat{=}} T^{-2}\sum_{t=1}^{T}(u_t\theta)(u_t\theta)', \tag{IA.10}$$

where $u_t = (Y'Y)^{-1}Y'\nu_t$, with ν_t the regression errors of the first-step estimation of $\widehat{\beta}$ and Y the regressor variable (either r_b or r_n).

C. Repeat Sales Method

This appendix contains details on the repeat sales method used to form returns on CDS portfolios. Let $k(i)$ be the portfolio that contains constituent i and let T be the number of periods in our sample. For constituent i, we assume that the spread quote of a CDS contract $p_{i,t}$ is given by

$$p_{i,t} = CDS_{k(i),t} + u_{i,t}, \tag{IA.11}$$

where $CDS_{k(i),t}$ is the portfolio spread level (which is to be estimated) and $u_{i,t}$ is a quote-specific error term, which has mean zero and constant variance σ_u and is uncorrelated with the other variables and its own lags. To illustrate the approach, suppose we have three transactions in constituent i, say at times s, s', and s'' with $s < s' < s''$. We can then specify spread innovations

$$\Delta p_{i,ss'} = p_{i,s'} - p_{i,s} = \sum_{j=2}^{T} x_{i,j,ss'} \Delta CDS_{k(i),j} + (u_{i,s'} - u_{i,s}) \tag{IA.12}$$

$$\Delta p_{i,s's''} = p_{i,s''} - p_{i,s'} = \sum_{j=2}^{T} x_{i,j,s's''} \Delta CDS_{k(i),j} + (u_{i,s''} - u_{i,s'}),$$

where $x_{i,j,ss'}$ is a dummy that defines whether $j \in [s, s']$. The error covariance matrix is given by

$$\mathrm{Var}(\Delta p_{i,ss'}) = 2\sigma_u^2, \ \mathrm{Var}(\Delta p_{i,s's''}) = 2\sigma_u^2, \ \mathrm{Cov}(\Delta p_{i,ss'}, \Delta p_{i,s's''}) = -\sigma_u^2. \tag{IA.13}$$

We can write our spread innovations for all constituents of $k(i)$ up to time T in matrix form as

$$\Delta p = x \Delta CDS_{k(i)} + v, \tag{IA.14}$$

where $v = \Delta u$. The best linear unbiased estimator of $\Delta CDS_{k(i)}$ is given by

$$\widehat{\Delta CDS}_{k(i)} = (x' M^{-1} x)^{-1} x' M^{-1} \Delta p, \tag{IA.15}$$

where M is the (sparse, block diagonal) covariance matrix of v. Empirically, σ_u is unknown. However, because M is known up to a scalar, which drops out, it turns out to be possible to consistently estimate $\Delta CDS_{k(i)}$ without knowledge of σ_u using regression.

D. Risk-free Rates and Default Probabilities

To construct excess returns from CDS spread changes, we need risk-free discount rates. Lando and Feldhütter (2008) argue that despite the AA default risk premium present in LIBOR rates, the best estimates of risk-free rates are obtained from swap rates. Therefore, we use daily data on the three-month LIBOR-based swap curve with a maturity of one up to six years. Swap rates are obtained from Datastream. To construct zero-coupon rates, we assume that these are piece-wise constant and subsequently bootstrap these rates from the observed term structure of swap rates.

To obtain the risk-neutral default probabilities, which are also needed to construct excess returns, we assume for simplicity that CDS spreads only reflect default risk, that the risk-neutral default intensity is constant over the maturity period, and that there is a deterministic loss rate $L = 60\%$. We then solve the CDS pricing equation under these assumptions to obtain the default intensity and compute the risk-neutral probabilities (Duffie and Singleton (2003)):

$$CDS_{k,t} = 4 \frac{L \sum_{j=1}^{(T-t)} \mathbb{Q}_{k,t}^{SV}(t+j-1) \mathbb{Q}_{k,t}^{def|SV}(t+j) B_t(t+j)}{\sum_{j=1}^{(T-t)} \mathbb{Q}_{k,t}^{SV}(t+j) B(t, t+j)}, \tag{IA.16}$$

$$\mathbb{Q}_{k,t}^{SV}(t+j) = \exp(-\lambda_{k,t} j), \ \mathbb{Q}_{k,t}^{def|SV} = 1 - \exp(-\lambda_{k,t}),$$

where $\mathbb{Q}_{k,t}^{def|SV}(t+j)$ is the risk-neutral probability of a default in period $t+j$ conditional on survival up to time $t+j-1$. We calculate these probabilities for each CDS portfolio and for each week in the empirical analysis.

Naturally, there is an inconsistency in assuming that CDS prices are only driven by default risk when the goal is to identify a nondefault component. However, if we iterate our estimation procedure, by correcting the CDS spread and λ for the estimated liquidity effect and reestimating the model, we find results that are extremely close to the results reported here. Note that we only need $\mathbb{Q}^{SV}$ to calculate excess returns.

E. References

Duffie, J. Darrell, and Kenneth J. Singleton, 2003, *Credit Risk: Pricing, Measurement and Management*. (Princeton University Press Princeton).

Lando, David, and Peter Feldhütter, 2008, Decomposing swap spreads, *Journal of Financial Economics* 88, 375–405.

Shanken, Jay, 1992, On the estimation of beta-pricing models, *Review of Financial Studies*, 5, 1–33.

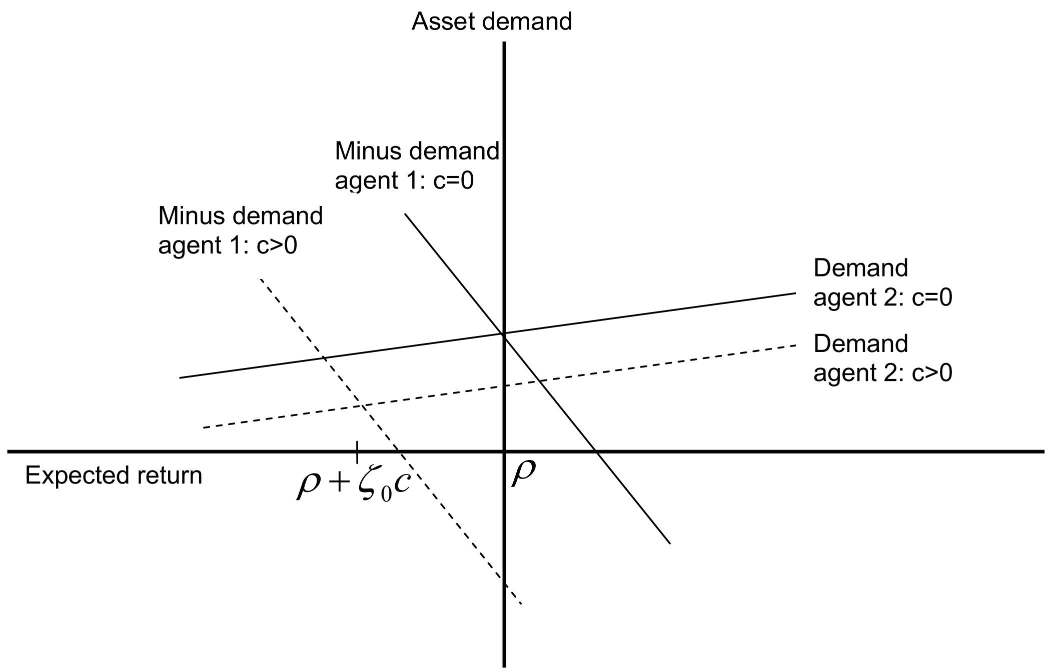

Figure IA.1

Asset pricing equilibrium with transaction costs: Example with negative effect of transaction costs on expected returns.

The figure illustrates the asset pricing equilibrium with constant liquidity costs, as in the example of Section I.D, for the case where the investor long in the asset (agent 2) is less aggressive than the investor who is short (agent 1). The figure graphs (minus) the asset demand of an investor with hedging needs ($-w_1 y_1$) who is short in the asset and an investor with no hedging demand ($w_2 y_2$) who is long. The solid lines reflect the situation without transaction costs; the dashed lines reflect the situation with transaction costs c. ρ is the equilibrium expected return without transaction costs; $\rho+\zeta_0 c$ is the equilibrium expected return with transaction costs.

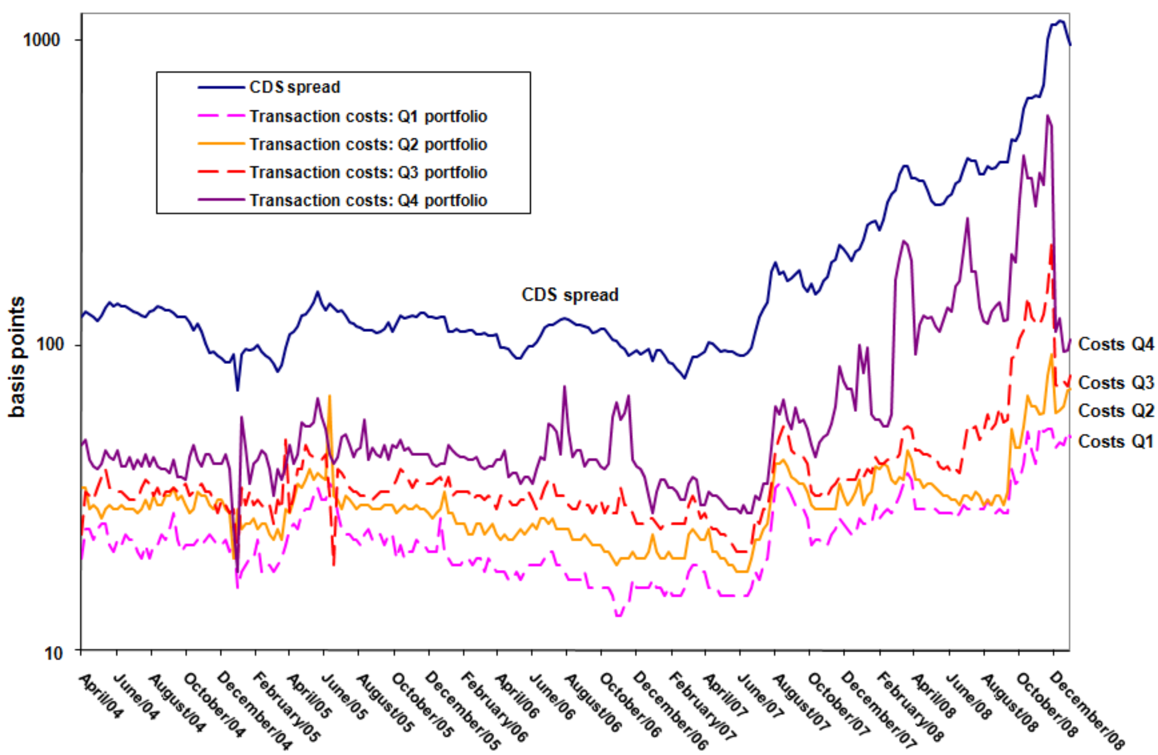

Figure IA.2

Time series of CDS spreads and bid-ask spreads.

The graph presents the time-series of weekly CDS spreads (geometric average across all CDS portfolios) from April 2004 to December 2008 on a log scale in annual basis points. Also included are time series of bid-ask spreads for four bid-ask spread quartiles. These are calculated from the sequential sort on credit rating and bid-ask spread, with bid-ask spreads averaged across portfolios with different credit ratings.

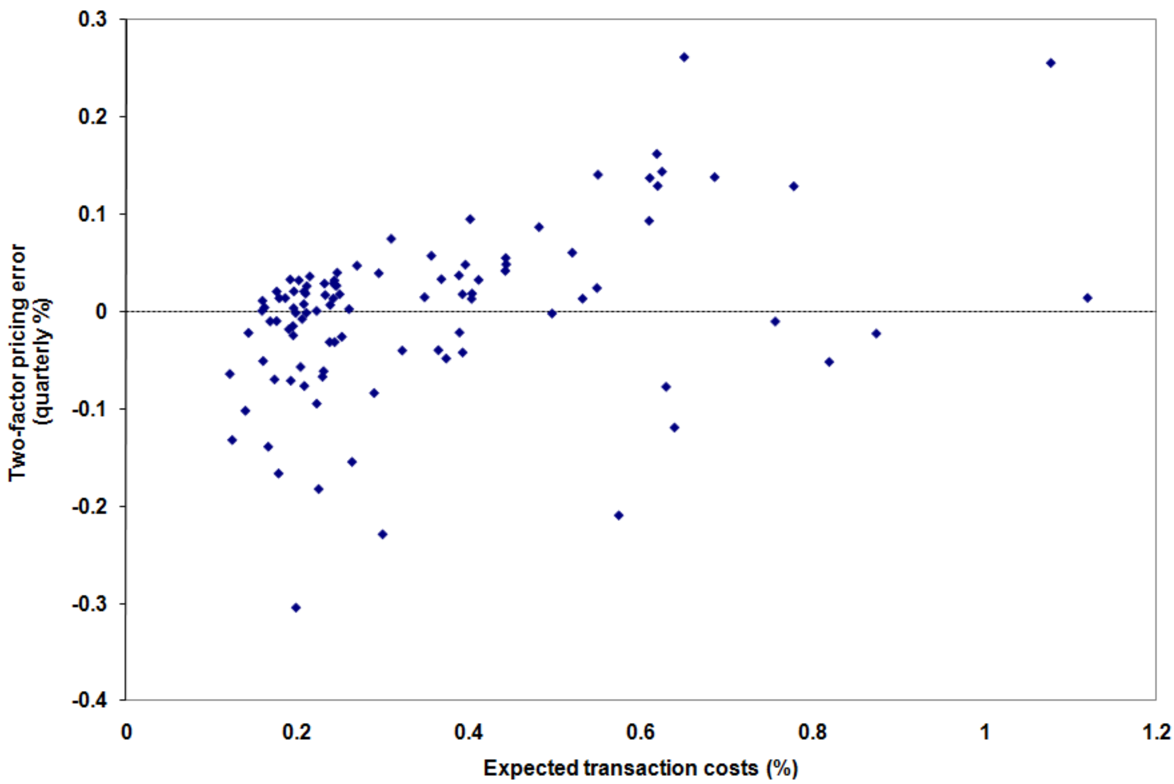

Figure IA.3

Two-factor pricing errors versus expected liquidity.

On the y-axis, the graph has pricing errors from a two-factor asset pricing model with equity market risk and systematic credit risk (PCA factor) as factors (specification (3) in Table IV), for 100 CDS portfolios (sorted first on credit rating, and then on leverage, total debt, total syndicated loan amount, bid-ask spread, or quote frequency). The x-axis has the expected liquidity of each portfolio (in percentage), calculated as the average of the weekly transaction costs of each portfolio. All returns are in percentages for a quarterly period.

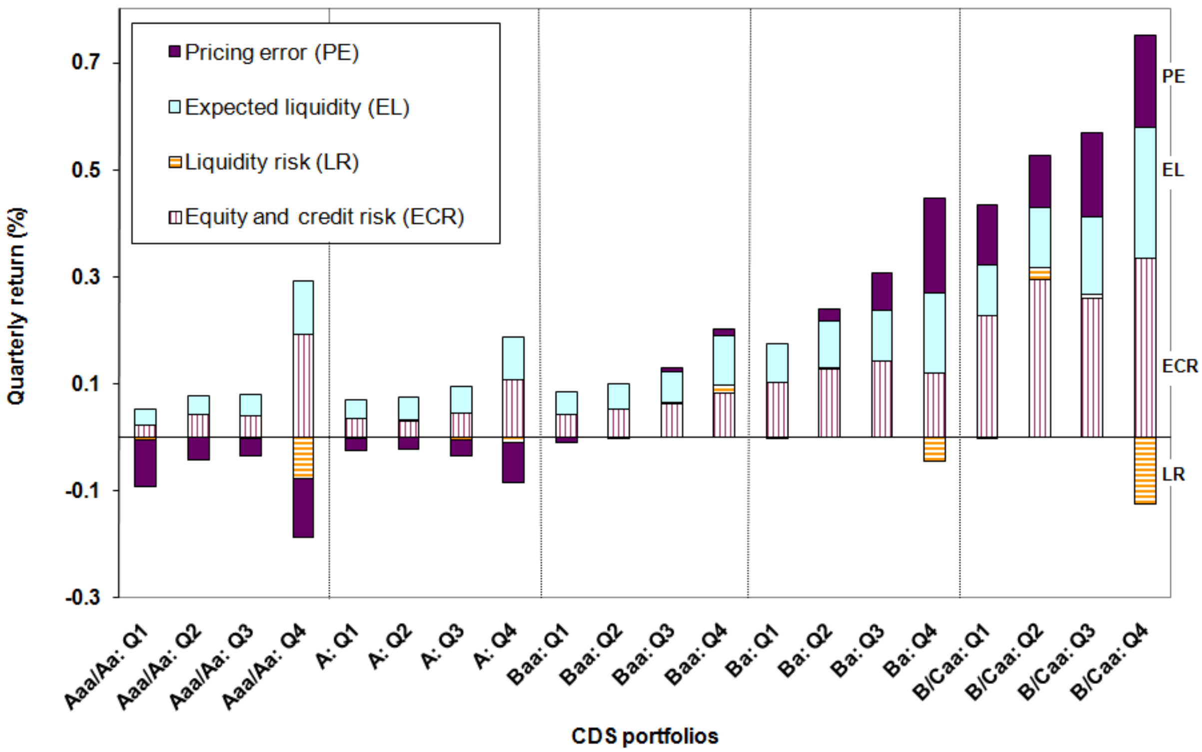

Figure IA.4

Decomposition of expected CDS returns: Model without intercept.

Using the GMM estimates of a model without intercept in Table V, specification (9) and the model for expected CDS returns in (16), the graph decomposes expected portfolio CDS returns into market risk premia (sum of equity risk premium and credit risk premium), expected liquidity, liquidity risk, and a pricing error. Results are presented for the sequential sort on credit rating (five categories) and bid-ask spread (four quartiles). All returns are in percentages for a quarterly period.