Adaptive Expectations and Stock Market Crashes

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Abstract

A theory is developed that explains how the stock market can crash in the absence of news about fundamentals, and why crashes are more common than frenzies. A crash occurs via the interaction of rational and naive investors. Naive traders believe in a simple (but reasonable) statistical model of stock prices: that prices follow a random walk with serially correlated volatility. They predict future volatility adaptively, as a weighted average of past squared price changes. From time to time, the rational traders sharply lower their demand for stocks, causing prices to fall below fundamentals. This raises naive investors' assessment of future volatility. Since naive traders are risk averse, their demand for stocks falls. This lowers the market's risk-bearing ability after the crash. Anticipating this, a rational trader has no incentive to bid up prices on the day of the crash. Unlike other explanations of market crashes, this mechanism is fundamentally asymmetric: the price of stocks cannot exceed fundamentals, so frenzies or bubbles cannot occur.

Keywords: Stock market crashes, adaptive expectations.

1 Introduction

On October 19, 1987, the S&P 500 index fell by 20.5%. Evidence from option prices suggests that investors expect more crashes to occur.[1] What causes such jumps in prices? The explanation should reflect the fact that many traders were responding to prior price declines themselves, rather than to news about the economy or firm profitability.[2] The theory should also explain why crashes happen more often than comparable-sized frenzies, in which prices rise sharply.[3]

We present a new theory in which a crash results from the interaction between rational and naive traders. The naive traders believe that stock prices follow a random walk with serially correlated volatility.[4] They predict future volatility adaptively, as a weighted average of recent squared price changes.[5] This contrasts with the rational traders, who predict future volatility correctly using knowledge of other players' strategies.

There is some historical justification for the idea that the presence of naive traders makes crashes more likely. The largest crashes, in 1929 and 1987, occurred after extended bull markets that attracted many inexperienced investors into the stock market. The investor Bernard Baruch wrote, in reference to the crash of 1929,

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[1] Ait-Sahalia, Yared, and Wang [2] find that the high prices of out-of-the-money put options on the S&P 500 index are inconsistent with a model in which stock prices change continuously according to a Markov process. However, they are consistent with occasional downward jumps in stock prices.

[2] According to Shiller's postcrash survey [37, p. 386], declining prices on October 14-16 and the morning of October 19 were the news items that most influenced investors' views of the stock market on October 19, 1987. See also Cutler, Poterba, and Summers [12] and Shiller [37, pp. 373-4].

[3] Nine out of the ten largest one-day price movements in the postwar period were declines (Hong and Stein [22]). Since 1945, the S&P Composite index has fallen by 5% or more on 13 separate days; the average of these declines was 7.5%. The index has risen by over 5% on only 5 days; the average was only 5.9%.

[4] "Volatility" refers to the variance of the change in stock prices.

[5] This model appears to be the first to explore how equilibrium prices are affected if some agents have adaptive expectations of future return volatility.

> Never before had there been such gambling as there was in those last turbulent years of the twenties; but few people realized they were gambling—they thought they had a sure thing. ... Taxi drivers told you what to buy. The shoeshine boy could give you a summary of the day's financial news as he worked with rag and polish. (Baruch [5, pp. 219-220])

In our model, a crash occurs in the following way. The rational traders observe a common signal that acts as a coordinating device. For certain values of this signal, they dump their shares. The resulting crash raises the naive traders' assessment of the risk in the market. Since naive traders are risk averse, they become less willing to own stocks. This lowers the market's risk-bearing capacity. Thus, the price remains lower for some time after the crash. Expecting this, rational traders have no incentive to bid up the stock price on the crash day. This makes the crash a self-fulfilling prophecy.

Importantly, this mechanism does not give rise to frenzies. If rational traders were all to *buy* shares one day instead of sell, the sharp price increase would also raise the naive traders' estimate of future volatility, prompting them to sell and pushing prices down in the following period. Anticipating this, each rational trader would have an incentive to sell when the others were buying. This is consistent with the empirical rarity of frenzies.[6]

This model captures other stylized facts surrounding crashes. Prices jump discontinuously. Some traders - the naive ones - sell in response to prior price changes. In addition, crashes are unexpected: until the crash signal is observed, no one knows a crash is about to happen. This mirrors findings of Bates [6] that option prices indicated no crash fears in the 2 months leading up to the 1987 crash.

The presence of naive traders is necessary for crashes to occur in our model. The crash makes the naive traders expect high volatility in the future, which reduces their willingness to own stock on the crash day. This lowers the risk-bearing capacity of the

[6] Naive traders in the model believe that prices follow a random walk. Thus, they do not believe that price increases will be followed by more increases. If they did believe this (à la the feedback traders of De Long, Shleifer, Summers, and Waldman [13]), frenzies might occur. However, the mechanism we describe would still reduce the sizes of frenzies relative to crashes.

market and thus makes a lower price necessary. If there were only rational agents, they would know that the crash was a transitory event and would thus prevent the crash by bidding prices up on the crash day. But while some naivete is needed for a crash to occur, it takes a very mild form: naive traders believe that stock prices follow a random walk with serially correlated volatility. Until recent years, this belief was the orthodox position of the economics community [3, 15, 30, 31].

In our theory, naive traders fare worse than rational traders. However, the usual criticism is not valid that naive traders cannot play a role in price dynamics since they will eventually be driven from the market. This is because crashes are rare. Most of the investors in the market during the 1987 crash had not been born in 1929.

In the model, the rational traders sell in response to a common signal that acts as a coordinating device. What played this role in 1987? On the morning of the crash on October 19, 1987, the *Wall Street Journal* published a chart suggesting a similarity between recent market action and stock prices in 1929.[7] This chart is reproduced in Figure 1.

This similarity is more than just casual. On the eve of the 1987 crash, the recent behavior of the Dow Jones Industrial Average (DJIA) was more similar to its behavior on the eve of the 1929 crash than at any Friday between the two dates. More precisely, we computed charts of the log closing Dow Jones Industrial Average over the 100 trading days ending on each Friday from 1930 through 1987. (We restrict to Fridays since both crashes occurred on a Monday.) We superimposed each of these charts on the corresponding chart for the Friday that preceded the 1929 crash. We then guaged the similarity between the two curves by summing the absolute differences between the log DJIA on each of the 100 days. This similarity was greater on the Friday preceding the 1987 crash than on any prior Friday in the 1930-1987 period.[8]

[7] This chart is discussed by Shiller [38].

[8] Let D_t be the log closing DJIA on day t. Let T be 10/25/1929, the Friday preceding the 1929 crash. The area between the 100-day charts ending on days t and T is given by $A_t = \min_x \sum_{i=0}^{99} |D_{t-i} - D_{T-i} - x|$. The adjustment of x corresponds to moving one chart vertically until this area is at a minimum. We measure A_t for t equal to each Friday from 1930 to 1987. The smallest

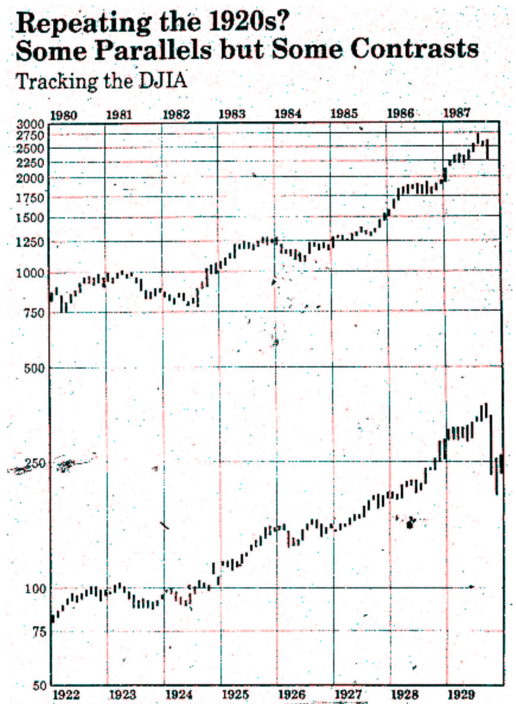

Figure 1:

This parallel was noticed independently by other investors. Stanley Druckenmiller, then manager of George Soros's Quantum Fund, states:

> That Friday [October 16, 1987] after the close, I happened to speak to Soros. He said that he had a study done by Paul Tudor Jones that he wanted to show me. [...] *The analysis [...] illustrated the extremely close correlation in price action between the 1987 stock market and the 1929 stock market, with the implicit conclusion that we were now at the brink of a collapse* [emphasis added]. I was sick to my stomach when I went home that evening. I realized that I had blown it and that the market was about to crash. [35, pp. 198-9]

Shiller argues that such reliance on historical parallels is an example of the *represen-*

A_t is 2.29 and occurred on the Friday that preceded the 1987 crash (10/16/1987). The next smallest, 2.53, was reached the prior Friday (10/9/1987). The average value of A_t from 1930 to 1987 (Fridays only) is 5.76 and the maximum is 23.38. (The New York Stock Exchange was open for two hours in the morning on Saturdays until 1952. In order to ensure a consistent relation between trading days and calendar time, we omit these Saturday index levels from our analysis.)

tativeness heuristic (Tversky and Kahneman [39]):[9]

> a tendency for people to categorize events as typical or representative of a well-known class, and then, in making probability estimates, to overstress the importance of such a categorization, disregarding evidence about the underlying probabilities. (Shiller [38, p. 22])

This quote defines the representativeness heuristic as irrational. Our theory suggests a rational reason for using the representativeness heuristic: it may serve as a coordinating device for some investors.

The rest of the paper is organized as follows. Relevant literature is reviewed in section 2. Section 3 explains the naive traders' beliefs. The model is presented in section 4 and solved in section 5.

2 Literature Review

Explaining crashes is a central problem in financial economics. Occasional crashes are an essential feature of the aggregate stock market in modern times and appear to be crucial for understanding the empirical patterns of option prices (see footnote 1). A satisfactory model of crashes should generate asymmetric price jumps from little or no fundamental news. By and large, prior models of crashes do not yield this phenomenon.

One group of crash models studies how large price changes can occur if small changes in the environment lead substantial information to be revealed to partially informed investors. This class of models includes Abreu and Brunnermeier [1], Caplin and Leahy [10], Hong and Stein [22], Kraus and Smith [26], Lee [27], Romer [34], and Zeira [40]. While these models yield price jumps with little or no fundamental news, they do not yield the prediction that crashes are more common than frenzies.[10]

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[9] Gilboa and Schmeidler [18, 19] offer "case-based decision theory," which may formally model this type of reasoning.

[10] In the two exceptions, plausible changes in the model yield frenzies rather than crashes. Abreu and Brunnermeier assume that investors overestimate the dividend growth rate; if investors were to

The models of Gennotte and Leland [17], Grossman [20], and Jacklin, Kleidon, and Pfleiderer [23], explore how rational investors can mistake the informational content of the trades of nonrational investors. These models assume the existence of portfolio insurers, who mechanically sell stocks when prices fall and buy when they rise. If rational traders underestimate the extent of this behavior, they will mistake it for informed trading. This can magnify the price effects of minor news.[11] The first two papers interpret the crash as coming from such a mistake. Jacklin *et al* interpret the price increase before the crash as coming from underestimation of portfolio insurance, while the crash itself occurred when informed traders realized their mistake.

The main drawback of these models is that they do not generate skewed returns: crashes and frenzies are equally likely. In addition, in most of these models, the crash is caused by a misinterpretation by rational investors. One would expect prices to recover quickly as this confusion is cleared up. In practice, prices returned to precrash levels only a year after the 1987 crash. According to our theory, prices can remain low if naive investors remain "crashophobic" after the crash. Indeed, evidence from option prices indicates that crash fears have been present in the years since 1987 but were not present before the crash (Jackwerth and Rubinstein [24]).

The model of Grossman and Zhou [21], while not aimed at explaining crashes, does yield some of their properties. They study a model with symmetric information and two types of risk averse investors who each maximize expected consumption utility. One type, the "portfolio insurers," have an additional constraint that their wealth must not

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underestimate this rate, there would be frenzies instead of crashes. Hong and Stein assume short-sale constraints; if these were replaced with margin constraints on leveraged buying, crashes would be replaced by frenzies. In contrast, to obtain frenzies in our model one would have to replace risk aversion by risk-seeking behavior, which is implausible.

[11] A related model is Barlevy and Veronesi [4], in which all investors are rational and some are uninformed. A price decline signals negative information to the uninformed investors, which lowers the price further, which signals the possibility of even worse information, and so on. Thus, crashes and frenzies can occur without assuming irrational or mistaken investors because of nonstandard assumptions about the signal distribution.

fall before a certain level. As fundamentals worsen, the portfolio insurers sell stock at an accelerating rate, leading to an increase in volatility. This model does not yield news-free price jumps.

Our theory is related to the "volatility feedback" effect first studied by French, Schwert, and Stambaugh [16], Malkiel [29] and Pindyck [32]. They point out that greater stock market volatility can lead to a higher risk premium and thus to lower stock prices.[12] Campbell and Hentschel [9] show that this effect can also give rise to negative skew: price declines are larger, on average, than price advances. They assume a fully rational representative agent who sees dividends that follow a process with serially correlated volatility. Large dividend shocks lead to lower prices since they indicate an increase in volatility and the agent is risk averse. This "volatility feedback effect" dampens the price effects of positive dividend news and exaggerates the price effects of negative dividend news.

While the model of Campbell and Hentschel generates negative skew, it does not give news-free jumps: prices are a continuous function of fundamentals. Indeed, in their calibrated model, the crash of 1987 results from a substantial negative dividend shock. This is inconsistent with the findings of Schiller and others (footnote 2) and the consensus among authors of prior crash models (discussed above) that the crashes of 1929 and 1987 were not caused by any fundamental news that was revealed around the crash date. One point of our model is that if some traders believe that *prices* display serially correlated volatility, then their presence in the market can yield news-free crashes without frenzies.

3 Naive Traders' Beliefs

Let p_t be the stock price in period t. The naive traders believe that prices follow a random walk (or more precisely, a Martingale): $E_t^{\text{Naive}}(p_{t+1}) = p_t$. Observing p_t, the

[12] In response to Pindyck [32], Poterba and Summers [33] produced evidence that volatility changes are not persistent enough to effect stock prices much. They model volatility as an AR(1) process. However, Chou [11] subsequently found much stronger persistence using GARCH, a more flexible specification.

naive traders believe that p_{t+1} will be normally distributed with mean p_t and variance V_t. They believe that the volatility V_t is serially correlated. Their prediction of future volatility is formed adaptively, as a weighted sum of the current period's squared price change and their prediction, in the prior period, of what volatility in the current period would be:

$$V_t = \alpha(p_t - p_{t-1})^2 + \beta V_{t-1} \tag{1}$$

for some fixed, positive constants α and β.

While we cite psychological evidence above to justify the behavior of naive traders, their beliefs are not completely unreasonable. For example, the view that prices follow a random walk dates from Bachelier [3] and has been widely promulgated. In his best-selling textbook, Sharpe [36, p. 315] writes:

> Stock returns exhibit almost no serial correlation: the particular value of return in the last period provides little if any help in predicting the likelihood of various possible returns in the next period.

Malkiel makes the same point forcefully in his well-known book *A Random Walk Down Wall Street* [30].

The view that volatility is serially correlated has been the consensus in academic circles since Mandelbrot [31, pp. 418-9] and Fama [15, pp. 85-7] and is taught in popular textbooks such as Brealey and Myers [8, p. 510] and Sharpe [36]. Sharpe also discusses how one can predict future volatility using historic volatility, and why it is worthwhile to put more weight on recent returns [36, p. 441]. Equation (1) is an example of this: by substituting repeatedly for V on the right hand side, one can express V_t as a geometric weighted sum of past squared returns:[13]

$$V_t = \alpha \sum_{i=0}^{\infty} \beta^i \left(p_{t-i} - p_{t-i-1}\right)^2 \tag{2}$$

[13] For Sharpe, "return" refers to the relative change in price, $\frac{p_t - p_{t-1}}{p_{t-1}}$, while our naive traders form beliefs about the absolute change. This makes no essential difference. If naive traders predict the variance of the relative change in price, the results are qualitatively the same but the notation is more cumbersome.

These views are also supported by casual empirical analysis. From day to day, the serial correlation of volatility is much stronger than the serial correlation of returns. For the S&P Composite Index from 1929 to 1999, the serial correlation of daily volatility was 0.23; in comparison, the serial correlation of daily returns was only 0.055.[14] For the Dow Jones Industrial Average over the same period, the analogous figures were 0.22 and 0.052, respectively. In addition, since daily returns on S&P futures are essentially uncorrelated,[15] in order to exploit the correlation of daily returns one would have to incur the high transaction costs of trading many individual stocks.

Adaptive expectations have also become the dominant approach among financial econometricians for modelling the dynamics of asset price volatility. In 1982, Engle [14] first proposed the ARCH (Autoregressive Conditional Heteroskedasticity) model, in which next period's volatility is a weighted sum of past realized volatilities. In 1986, Bollerslev [7] generalized this to GARCH (Generalized ARCH) by letting next period's volatility depend also on past predicted volatilities. In the past two decades, over 200 journal articles have used ARCH or GARCH to model the changing volatility of asset returns.[16] Equation (1) is, indeed, a simple GARCH model.

4 The Model

The game takes place in three periods: $t = 0, 1, 2$. There is a measure μ of rational traders and $1 - \mu$ of naive traders. Agents consume only in period 2; they maximize expected utility $EU(W_2) = E\left[-e^{-\lambda W_2}\right]$, where W_t is wealth in period t and λ is the

[14] These statistics are based on the usual definition of the ex-dividend return, $r_t = \frac{p_t - p_{t-1}}{p_{t-1}}$. The serial correlation of returns is the sample correlation of r_t with r_{t-1}; the serial correlation of volatility is the sample correlation of r_t^2 with $r_{t=1}^2$.

[15] Evidence for this comes from MacKinlay and Ramaswamy [28], who compute daily autocorrelations in log returns for the S&P 500 index and for futures contracts on this index during the 1983-1987 period. They find an average autocorrelation of 6.04% for daily index returns versus -0.24% for daily futures returns [28, p. 148, panel E of table 2].

[16] Author's tabulation from Econlit.

coefficient of absolute risk aversion. There are two assets: one ("stocks"), pays an i.i.d. dividend $\delta_t \sim N(\overline{\delta}, \sigma_\delta^2)$ per share[17] after the market closes in each period $t = 0, 1$, and a fixed liquidating dividend of D in period 2. The other asset, bonds, is in infinitely elastic supply and pays interest at a fixed net rate of r after the market closes in periods 0 and 1.

Let p_t be the price of a share of stock in period t. If an agent buys x_t shares of stock in period $t = 0, 1$, costing her $p_t x_t$, her wealth in period $t + 1$ is

$$W_{t+1} = x_t \left(p_{t+1} + \delta_t\right) + (W_t - p_t x_t)(1 + r) \tag{3}$$

We make two assumptions to guarantee the existence of a constant-price equilibrium, in which $p_0 = p_1 = D$. The first is to fix D at $\frac{1}{r}\left[\overline{\delta} - \lambda\sigma_\delta^2\right]$. The second is to fix the number of shares of stock at $\frac{1}{1+r}$ in period zero and one in period 1. Why? Agents in period 1 face a one-period problem with objective function $e^{-\lambda W_2}$. But agents in period 0 invest as if they are more risk averse. Their objective function is proportional to $e^{-\lambda(1+r)W_1}$: by (3), W_2 equals $W_1(1+r)$ plus a term that is independent of W_1.[18] As a result, stock demand in any constant-price equilibrium must be lower in period 0 than in period 1. For markets to clear, we must assume that supply is also initially lower.

The sequence of events is as follows.

Period 0. No signals are observed and all traders trade. The role of this period is to permit optimal risk-sharing and to establish a base price p_0 for the stock. At the end of the period, the dividend δ_0 per share is announced and distributed. The naive traders' prediction V_0 of the variance of the initial price p_0 is taken to be zero.

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[17] The assumption of i.i.d. dividends implies that there is no fundamental news that is relevant to the stock price. This stylized assumption is made to show that crashes can occur without any fundamental news. Serially correlated dividend shocks, while perhaps making the model more realistic, would obscure this point without essentially changing the results.

[18] Since returns in period 2 are normally distributed, the amount the agent invests in stock in period 1 does not depend on her wealth W_1.

Period 1. With probability ε, the rational traders all observe a crash signal; with probability $1-\varepsilon$, no signal is seen. The crash signal is a pure coordinating device. All traders then trade at some price p_1. Finally, the dividend δ_1 is announced and paid.

Period 2. The liquidating dividend D is paid. There is no trade in this period.

When trade takes place, naive agents simultaneously submit demand functions: the quantity of shares they wish to buy at each price. Each rational trader submits a single limit order of the form "I will buy x shares if the price per share is no greater than p."[19] Since there is a continuum of agents, they will act as price takers. The market-clearing price is determined by the condition that the demand for stocks equal the unit supply:

$$\mu x_0^R + (1-\mu)x_0^N = \frac{1}{1+r} \tag{4}$$

$$\mu x_1^R + (1-\mu)x_1^N = 1 \tag{5}$$

where x_t^R and x_t^N are the time-t stock demands of rational and naive traders, respectively.

5 Results

We first solve for investors' stock demand functions. We will make use of the following well known property (proof available on request).

Lemma 1 *Suppose an agent has wealth W and the share price is p. The agent buys x shares of a risky asset that pays a gross return R in the next period and invests the rest of her wealth in a riskless asset that pays gross return $1+r$. Let the agent's*

[19] By making limit orders, the rational traders collectively determine whether a crash will occur by picking a particular point on their demand curves. This permits decentralized crashes: many different investors suddenly deciding to pay less for stock since they *predict* that the price will be lower. If rational agents were to submit demand curves, crashes could still occur. However, since there is no uncertainty about the liquidating dividend, a rational agent's demand curve in period 1 would be the same regardless of whether or not a crash signal is seen. Crashes would be centralized: they would take the form of a Walrasian auctioneer's choosing a low market-clearing price.

wealth in the next period be $W' = W(1+r) + x(R-(1+r)p)$. The agent seeks to maximize the expectation of $-e^{-\lambda W'}$. Assume the agent believes that $R \sim N(\mu, \sigma^2)$. If the agent behaves optimally, she will buy $\frac{\mu-(1+r)p}{\lambda\sigma^2}$ shares and her expected utility is $-\exp\left[-\lambda W(1+r) + \frac{(\mu-(1+r)p)^2}{\sigma^2}\right]$.

In period 1, rational agents expect each share to yield the gross return $D+\delta_1$, which is normally distributed with mean $D+\overline{\delta}$ and variance σ_δ^2. Their demand is thus:

$$x_1^R = \frac{D+\overline{\delta}-(1+r)p_1}{\lambda\sigma_\delta^2} \tag{6}$$

which is conventionally decreasing in the price p_1. Naive traders expect each share to yield the gross return $p_2+\delta_1$, which they believe is normally distributed with mean $p_1+\overline{\delta}$ and variance $V_1+\sigma_\delta^2$ where $V_1 = \alpha(p_1-p_0)^2$. (We assume, for simplicity, that naive traders' initial variance estimate V_0 is zero.) Naive traders' demand must equal:

$$x_1^N = \frac{\overline{\delta}-rp_1}{\lambda(\alpha(p_1-p_0)^2+\sigma_\delta^2)} \tag{7}$$

Importantly, a fall in p_1 below the initial price level p_0 raises the denominator: naive traders' estimate of future volatility rises, lowering their demand. This is the mechanism that gives rise to crashes.

In period 0, naive traders expect each share to yield the gross return $p_1+\delta_0$, which they believe is normally distributed with mean $p_0+\overline{\delta}$ and variance σ_δ^2. Naive traders' demand must equal:

$$x_0^N = \frac{\overline{\delta}-rp_0}{\lambda(1+r)\sigma_\delta^2} \tag{8}$$

Rational traders' demand cannot be computed explicitly since there is no reason to believe that p_1 will be normally distributed. However, we can compute in a constant-price equilibrium, in which $p_1 = p_0$ for sure. In this case, rational traders have the same beliefs as naive traders, so their demands are also given by

$$x_0^R = \frac{\overline{\delta}-rp_0}{\lambda(1+r)\sigma_\delta^2} \tag{9}$$

We first show that there is only one constant-price equilibrium:

Proposition 2 *There is only one constant-price equilibrium. In this equilibrium, the price always equals the variance-free price* $\overline{p} = D = \frac{1}{r}\left[\overline{\delta} - \lambda\sigma_\delta^2\right]$.

Proposition 3 shows that even if the price is not a constant, it can never exceed its fundamental level of $\overline{p}$. Thus, frenzies cannot occur in this model.

Proposition 3 *In any equilibrium, the stock price can never exceed* $\overline{p}$.

The intuition for Proposition 3 is as follows. In a given equilibrium, let $p^{\max}$ be the maximum price that can be attained in any period. When the price is $p^{\max}$, rational traders must expect the next period's return to be zero or negative. Naive traders, by assumption, expect it to be zero. Agents' expected returns in the constant price equilibrium are at least as optimistic as this; furthermore, agents in the constant-price equilibrium expect zero volatility. So if the price is $p^{\max}$, stocks cannot offer a more attractive return distribution to either type of agent than in the constant-price equilibrium. But then no agent will ever be willing to pay more than the price in that equilibrium, which is $\overline{p}$; hence, $p^{\max} \leq \overline{p}$.

We now consider equilibria in which there are two possible prices in period 1: a high price and a lower "crash" price that occurs with some small probability. A crash can occur since a price decline in period 1 raises the risk premium of naive traders. If α is large enough, this effect lowers naive traders' demand. This requires rational traders to hold a larger share of stock. Since there is dividend risk in period 1, the rational traders' risk premium for a marginal share rises, making a lower price consistent with equilibrium in period 1.

The following Proposition formalizes this, giving sufficient conditions for there to be an equilibrium with a positive probability of a crash. Importantly, crashes can occur for any positive proportion of naive agents.[20]

[20] We cannot compute rational traders' period-0 demand explicitly in a crash equilibrium since p_1 is not normally distributed. Our approach is instead first to give conditions under which a crash price could clear the market in period 1 if rational traders did not anticipate the crash; in this case their period-0 demand is given conveniently by (9). We then use the implicit function theorem to show that

Proposition 4 *The following properties hold generically. Fix* $\overline{\delta} > 0$, $\lambda > 0$, $\sigma_\delta^2 > 0$, $\mu \in (0,1)$, *and* $r > 0$, *such that*

1. *the variance-free price* $\overline{p} = (\overline{\delta} - \lambda\sigma_\delta^2)/r$ *is positive;*

2. *for a low enough positive price* p_1, *rational traders buy all the stock:* $\mu\frac{D+\overline{\delta}}{\lambda\sigma_\delta^2} > 1$.

Then there is a threshold α^* *such that for all* $\alpha > \alpha^*$, *for any small enough crash risk* $\varepsilon > 0$ *there are prices* p_0, p_1^L, *and* p_1^H, *such that the following is an equilibrium:*

1. *The period 0 price equals* p_0.

2. *With probability* ε, *the price equals* p_1^L *in period 1; otherwise it equals* p_1^H.

As $\varepsilon \to 0$, p_0 *and* p_1^H *both converge to* $\overline{p}$, *but* p_1^L *converges to a price that is strictly lower than* $\overline{p}$.

6 Simulations

A computed example shows that crashes can occur even if naive traders put a reasonably low weight on recent volatility in updating their beliefs. An example in this weight, α, equals 0.3 appears in Figure 2. The chart shows excess demand (demand minus supply) for stocks in period 1. The horizontal axis shows the ratio p_1/p_0 and the vertical axes gives excess demand; equilibrium in period 1 requires that this excess demand be zero.[21] Since excess demand crosses the zero axis in two places for these parameters, crashes can be either large (about 18% of the precrash price) or small (about 9%). Another example, with α lower, appears in figure 3.

The proportion of naive traders must be fairly high to sustain crashes - in these simulations, they are 80% and 83%, respectively. This results from the absence, in our

there must also be equilibria in which rational traders correctly anticipate that a crash will occur with a sufficiently small probability.

[21] The chart depicts the limit as the crash risk goes to zero. It is approximately correct when the crash risk is small but nonzero.

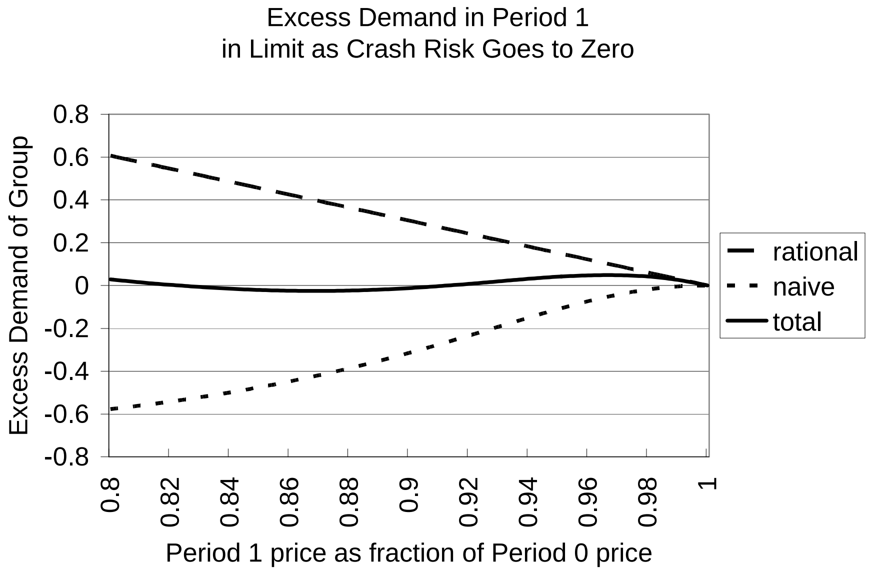

Figure 2: Example #1. Chart shows excess demand for stock, broken down into excess demand coming from rational and naive traders. Each curve shows not excess demand per investor but rather excess demand for the group. The horizontal axis is p_1/p_0. Parameters are $\alpha = 0.3$, $\mu = .2$, $\overline{\delta} = 1.15$, $\lambda = \sigma_{\delta}^2 = 1$, and $r = 1\%$.

model, of realistic constraints on margin buying. We now introduce margin constraints that take a simple form: in every period, a trader can invest up to 1.5 times her wealth in stocks. Initial aggregate wealth is set equal to the aggregate value of corporate stock; each trader is equally wealthy in period 0. A simulation appears in Figure 4. Naive traders' update sensitivity is moderate ($\alpha = 0.1$) and a majority (60%) of traders are rational. Despite this, there is a small but positive chance that prices will fall by 38% in period 1.

A Discussion

From time to time, stock indices have jumped by several percentage points in a single day. These jumps tend to be negative and they do not appear to be driven by public news about fundamentals. While these jumps are infrequent, evidence from derivatives

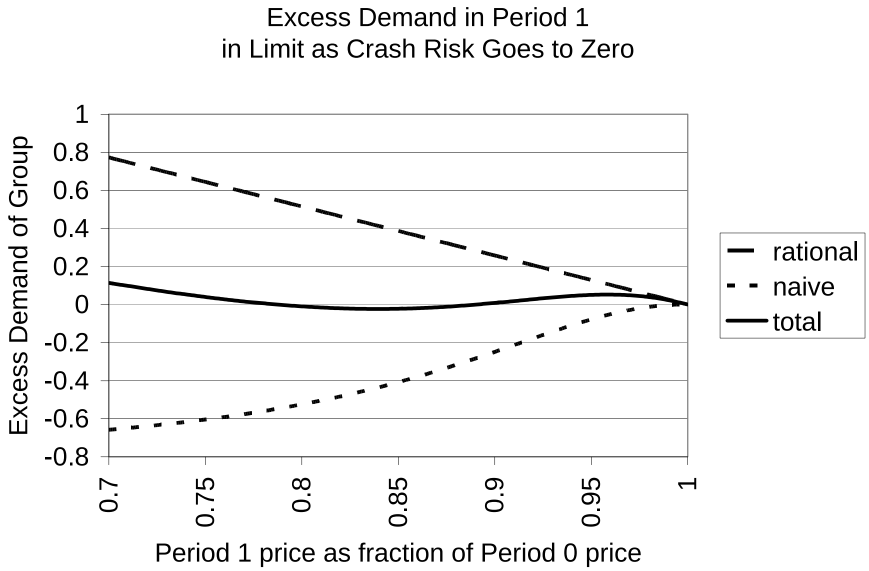

Figure 3: Example #2. Parameters are $\alpha = 0.2$, $\mu = 0.17$, $\overline{\delta} = 1.15$, $\lambda = \sigma_\delta^2 = 1$, and $r = 1\%$.

markets suggests that their anticipation has a significant effect on asset prices. By extension, these "crash fears" may raise the cost of corporate capital and depress economic growth.

For these reasons, it is important to understand the mechanisms that underlie crashes. By and large, the existing theoretical literature does not deliver. Models such as Campbell and Hentschel [9] that evince negative skew require large shocks to fundamentals; other models that yield news-free jumps do not generate negative skew. In the few models that do deliver both phenomena, the negative skew is due to special assumptions of the model that can be plausibly reversed, yielding positive skew. This paper presents a theory that does not appear to have this weakness. For example, positive skew could be generated by assuming investors are risk-loving, but this is not plausible.

The basic idea of the model is that some investors are naive: rather than taking into account the strategic behavior of other agents, they believe that stock prices follow a random walk with serially correlated volatility. This belief is approximately true in an empirical sense and was the mainstream view in the finance community for decades.

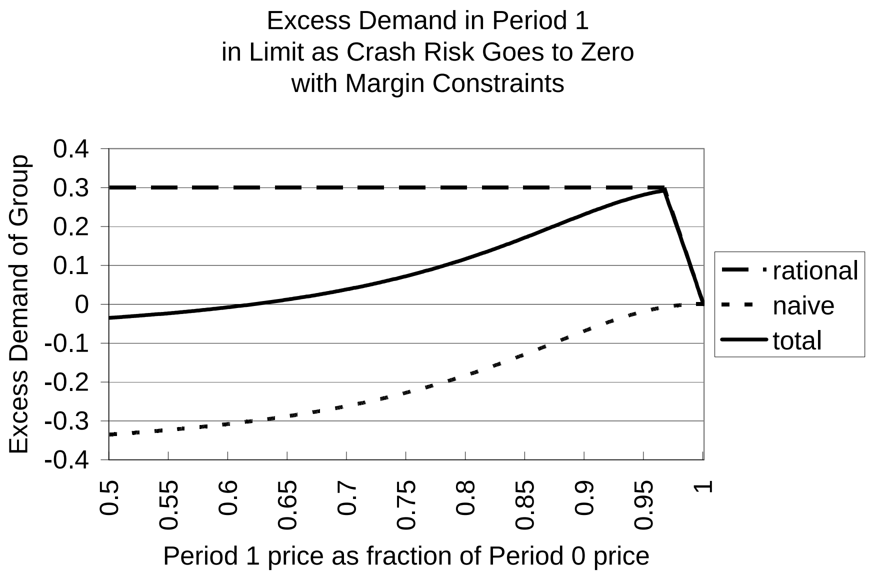

Figure 4: Example with margin constraints. Parameters are $\alpha = 0.1$, $\mu = 0.6$, $\overline{\delta} = 1.15$, $\lambda = \sigma_\delta^2 = 1$, and $r = 1\%$. Each trader can invest up to 1.5 times her wealth in stock in any period. All traders start with an equal share of total wealth, which initially equals the aggregate value of corporate stock.

In this simple model, we show that prices cannot exceed fundamentals but they can suddenly fall significantly below fundamentals. Simulations show that this can occur with reasonable updating by naive traders. Moreover, if realistic margin constraints are imposed, naive traders can also constitute a minority of investors in the market.

B Proofs

Proof of Proposition 2. In a constant-price equilibrium, $p_0 = p_1$; by (5), (6), and (7), market clearing in period 1 implies $p_1 = D = \frac{1}{r}\left[\overline{\delta} - \lambda\sigma_\delta^2\right]$. By Lemma 1, rational traders' demand equals $x_0^R = \frac{D+\overline{\delta}-(1+r)p_0}{\lambda(1+r)\sigma_\delta^2}$, so by (4) and (8) the market clears in period 0 if and only if p_0 also takes this value. Q.E.D.$_{\text{Proposition } \mathbf{2}}$

Proof of Proposition 3: First suppose $p_1 > \overline{p}$. By (7), $x_1^N \leq \frac{\overline{\delta}-rp_1}{\lambda\sigma_\delta^2} < \frac{\overline{\delta}-r\overline{p}}{\lambda\sigma_\delta^2} = 1$. So by (5), $x_1^R > 1$. However, by (6),

$$x_1^R = \frac{D + \overline{\delta} - (1+r)p_1}{\lambda \sigma_\delta^2} > 1$$
$$\Longrightarrow p_1 < \frac{1}{r}\left[\overline{\delta} - \lambda \sigma_\delta^2\right] = \overline{p}$$

a contradiction. Now suppose $p_0 > \overline{p}$. By (8), $x_0^N < \frac{1}{1+r}$, so by (4), $x_0^R > \frac{1}{1+r}$. Suppose rational traders believe that the distribution of prices in period 1 is given by the c.d.f. F, where by the preceding result $F(\overline{p}) = 1$. By Lemma 1, x_0^R maximizes

$$E_{p_1,\delta_1}\left[-\exp\left(\begin{array}{c} -\lambda\left(\begin{array}{c} W_0(1+r) \\ +x_0^R\left(p_1 + \delta_1 - (1+r)p_0\right)\end{array}\right)(1+r) \\ +\frac{\left(D+\overline{\delta}-(1+r)p_1\right)^2}{\sigma_\delta^2}\end{array}\right)\right]$$

or, factoring out a constant and reorganizing, maximizes[22]

$$E_{p_1,\delta_1}\left[-\exp\left(\begin{array}{c} -\lambda x_0^R(1+r)\delta_1 \\ -\lambda x_0^R\left(p_1 - (1+r)p_0\right)(1+r) + \frac{\left(D+\overline{\delta}-(1+r)p_1\right)^2}{\sigma_\delta^2}\end{array}\right)\right]$$
$$= E_{p_1}\left[-\exp h(x_0^R, p_1, p_0)\right]$$

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[22]For $z \sim N(\overline{z}, \sigma^2)$ and constants $a = \lambda x_0^R(1+r)$ and

$$b = -\lambda x_0^R\left(p_1 - (1+r)p_0\right)(1+r) + \frac{\left(D+\overline{\delta}-(1+r)p_1\right)^2}{\sigma_\delta^2}$$

we have

$$E\exp(-az+b) = \frac{1}{2\pi\sigma}\int_{z=-\infty}^{\infty}\exp\left(-az+b-\frac{(z-\overline{z})^2}{2\sigma^2}\right)dz$$
$$-az+b-\frac{(z-\overline{z})^2}{2\sigma^2} = b - \frac{1}{2\sigma^2}\left(2\sigma^2 az + z^2 - 2z\overline{z} + \overline{z}^2\right)$$
$$= b - \frac{1}{2\sigma^2}\left(z^2 - 2\left(\overline{z} - \sigma^2 a\right)z + \overline{z}^2\right)$$
$$= b - \frac{1}{2\sigma^2}\left[\left(z - \left(\overline{z} - \sigma^2 a\right)\right)^2 - \left(\overline{z} - \sigma^2 a\right)^2 + \overline{z}^2\right]$$
$$= b - \frac{1}{2\sigma^2}\left[\left(z - \left(\overline{z} - \sigma^2 a\right)\right)^2 - \overline{z}^2 + 2\sigma^2 a\overline{z} - \sigma^4 a^2 + \overline{z}^2\right]$$
$$= b - \frac{1}{2\sigma^2}\left[\left(z - \left(\overline{z} - \sigma^2 a\right)\right)^2 + 2\sigma^2 a\overline{z} - \sigma^4 a^2\right]$$
$$\Longrightarrow E\exp(-az+b) = \exp\left(b - a\overline{z} + \frac{\sigma^2 a^2}{2}\right)$$

where

$$h(x_0^R, p_1, p_0) = \left(\begin{array}{c} -\lambda x_0^R \left(p_1 - (1+r)p_0\right)(1+r) + \frac{\left(D+\overline{\delta}-(1+r)p_1\right)^2}{\sigma_\delta^2} \\ -\lambda x_0^R(1+r)\overline{\delta} + \frac{\sigma_\delta^2\left(\lambda x_0^R(1+r)\right)^2}{2} \end{array} \right)$$

However,

$$\begin{aligned} \frac{\partial^2}{\partial p_1 \partial x_0^R}\left[-\exp h(x_0^R, p_1, p_0)\right] &= -\frac{\partial}{\partial x_0^R}\left[(\exp h(\cdot))\frac{\partial}{\partial p_1}h(\cdot)\right] \\ &= -\exp h(\cdot)\left[\frac{\partial}{\partial x_0^R}h(\cdot)\frac{\partial}{\partial p_1}h(\cdot) + \frac{\partial^2}{\partial p_1 \partial x_0^R}h(\cdot)\right] \\ &= \exp h(\cdot)\lambda(1+r)\left[\begin{array}{c} \lambda(1+r)\left(\begin{array}{c} (1+r)p_0 - p_1 \\ +\sigma_\delta^2\lambda x_0^R(1+r) - \overline{\delta} \end{array} \right) \\ * \left(x_0^R + 2\frac{D+\overline{\delta}-(1+r)p_1}{\lambda\sigma_\delta^2}\right) + 1 \end{array} \right] \end{aligned}$$

We know that $p_1 \leq \overline{p}$. By assumption $p_0 > \overline{p}$, so $(1+r)p_0 - p_1 > 0$. If $x_0^R > \frac{1}{1+r}$, then

$$(1+r)p_0 - p_1 + \sigma_\delta^2\lambda x_0^R(1+r) - \overline{\delta} > r\overline{p} + \sigma_\delta^2\lambda - \overline{\delta} = 0$$

Moreover, in equilibrium rational trader demand in period 1 must be positive for the market to clear by (5), (6), and (7). Thus, by (6), $x_0^R + 2\frac{D+\overline{\delta}-(1+r)p_1}{\lambda\sigma_\delta^2} = x_0^R + 2x_1^R > 0$. This proves that $\frac{\partial^2}{\partial p_1 \partial x_0^R}\left[-\exp h(x_0^R, p_1, p_0)\right] > 0$ for any $x_0^R > \frac{1}{1+r}$. Since $p_1 \leq \overline{p}$, if we replace the true distribution of p_1 with a point mass on $\overline{p}$, the optimal x_0^R cannot fall. But by Lemma 1, x_0^R becomes $\frac{D+\overline{\delta}-(1+r)p_0}{\lambda(1+r)\sigma_\delta^2}$, which is less than $\frac{1}{1+r}$ if $p_0 > \overline{p}$. This is a contradiction. Q.E.D.$_{\text{Proposition 3}}$

Proof of Proposition 4: We cannot explicitly compute rational agents' demands in period 0 in this case, since returns are not normally distributed. We will overcome this by using instead their period-0 demand for the case in which they expect the price in period 1 to equal the variance-free price for sure. Under this assumption, there can be two market clearing prices in period 1: the variance-free price and a lower price, which is unanticipated in period 0. We will then show that there is continuity: for any small probability ε, there is an equilibrium in which the price is close to the lower price in period 1 with probability ε and close to the variance-free price with probablity $1-\varepsilon$.

If rational traders expect the price in period 1 to be the variance-free price, their demand in period zero is

$$\frac{\overline{p} + \overline{\delta} - (1+r)p_0}{\lambda(1+r)\sigma_\delta^2}$$

by Proposition 2. Hence, market-clearing in period 0 implies

$$\mu \frac{\overline{p} + \overline{\delta} - (1+r)p_0}{\lambda(1+r)\sigma_\delta^2} + (1-\mu)\frac{\overline{\delta} - rp_0}{\lambda(1+r)\sigma_\delta^2} = \frac{1}{1+r} \tag{10}$$

The unique solution is $p_0 = \overline{p}$: the variance-free price clears the market in period 0 as well.

Now suppose that we have reached period 1. By (5), (6), and (7), market clearing in period 1 requires that

$$\mu \frac{D - p_1 + \overline{\delta} - rp_1}{\lambda\sigma_\delta^2} + (1-\mu)\frac{\overline{\delta} - rp_1}{\lambda\left(\alpha(p_1 - \overline{p})^2 + \sigma_\delta^2\right)} = 1 \tag{11}$$

$p_1 = \overline{p}$ is always a solution to (11). Since the denominator in rational traders' demand is a strictly positive constant, by taking α arbitrarily large (possibly greater than 1), we can guarantee that the left hand side of (11) is negative for p_1 less than but sufficiently close to $\overline{p}$. But then by condition 2, by lowering $p_1 > 0$ far enough the left hand side of (11) becomes positive. Thus, for any sufficiently high α there is an equilibrium in which $p_0 = p_1 = \overline{p}$ for sure but there also exists another price p_1', strictly lower than $\overline{p}$, that would also clear the market in period 1.

We now prove that for sufficiently small crash risk $\varepsilon > 0$, there are equilibria that are close to this equilibrium. There are five variables: the crash probability, ε; rational traders' demand in period 0, x_0^R; the period 0 price p_0; the low period 1 price, p_1^L; the high period 1 price p_1^H. The equations for an equilibrium are:

Optimality of Rational Traders' Demand

$$0 = f^1(\varepsilon, x_0^R, p_0, p_1^L, p_1^H)$$

$$= \frac{\partial}{\partial x_0^R}\left\{\begin{array}{c} \varepsilon E_{\delta_1}\left[-\exp\left(\begin{array}{c}-\lambda\left(\begin{array}{c}W_0(1+r)\\ +x_0^R\left(p_1^L+\delta_1-(1+r)p_0\right)\end{array}\right)(1+r)\\ +\frac{\left(D+\overline{\delta}-(1+r)p_1^L\right)^2}{\sigma_\delta^2}\end{array}\right)\right] \\ +(1-\varepsilon)E_{\delta_1}\left[-\exp\left(\begin{array}{c}-\lambda\left(\begin{array}{c}W_0(1+r)\\ +x_0^R\left(p_1^H+\delta_1-(1+r)p_0\right)\end{array}\right)(1+r)\\ +\frac{\left(D+\overline{\delta}-(1+r)p_1^H\right)^2}{\sigma_\delta^2}\end{array}\right)\right]\end{array}\right\}$$

by Lemma 1;

Market Clearing in Period 0

$$0 = f^2(\varepsilon, x_0^R, p_0, p_1^L, p_1^H) = \mu x_0^R + (1-\mu)\frac{\overline{\delta} - rp_0}{\lambda(1+r)\sigma_\delta^2} - 1$$

Market Clearing in Period 1: Low Price

$$0 = f^3(\varepsilon, x_0^R, p_0, p_1^L, p_1^H) = \mu\frac{D - p_1^L + \overline{\delta} - rp_1^L}{\lambda\sigma_\delta^2} + (1-\mu)\frac{\overline{\delta} - rp_1^L}{\lambda\left(\alpha(p_1^L - p_0)^2 + \sigma_\delta^2\right)} - 1$$

Market Clearing in Period 1: High Price

$$0 = f^4(\varepsilon, x_0^R, p_0, p_1^L, p_1^H) = \mu\frac{D - p_1^H + \overline{\delta} - rp_1^H}{\lambda\sigma_\delta^2} + (1-\mu)\frac{\overline{\delta} - rp_1^H}{\lambda\left(\alpha(p_1^H - p_0)^2 + \sigma_\delta^2\right)} - 1$$

As shown above, one solution to this system is $(\varepsilon, x_0^R, p_0, p_1^L, p_1^H) = (0, x_0^R, \overline{p}, p_1', \overline{p})$. It is easy to verify that the functions f^n are continuously differentiable. In addition,

$$\det\begin{bmatrix} \frac{\partial f^1}{\partial x_0^R} & \frac{\partial f^1}{\partial p_0} & \frac{\partial f^1}{\partial p_1^L} & \frac{\partial f^1}{\partial p_1^H} \\ \frac{\partial f^2}{\partial x_0^R} & \frac{\partial f^2}{\partial p_0} & \frac{\partial f^2}{\partial p_1^L} & \frac{\partial f^2}{\partial p_1^H} \\ \frac{\partial f^3}{\partial x_0^R} & \frac{\partial f^3}{\partial p_0} & \frac{\partial f^3}{\partial p_1^L} & \frac{\partial f^3}{\partial p_1^H} \\ \frac{\partial f^4}{\partial x_0^R} & \frac{\partial f^4}{\partial p_0} & \frac{\partial f^4}{\partial p_1^L} & \frac{\partial f^4}{\partial p_1^H} \end{bmatrix} = \det\begin{bmatrix} \frac{\partial f^1}{\partial x_0^R} & \frac{\partial f^1}{\partial p_0} & \frac{\partial f^1}{\partial p_1^L} & \frac{\partial f^1}{\partial p_1^H} \\ \frac{\partial f^2}{\partial x_0^R} & \frac{\partial f^2}{\partial p_0} & 0 & 0 \\ 0 & \frac{\partial f^3}{\partial p_0} & \frac{\partial f^3}{\partial p_1^L} & 0 \\ 0 & \frac{\partial f^4}{\partial p_0} & 0 & \frac{\partial f^4}{\partial p_1^H} \end{bmatrix}$$

is generically nonzero. By the Implicit Function Theorem, there are unique, continuously differentiable functions $x_0^R(\varepsilon)$, $p_0(\varepsilon)$, $p_1^L(\varepsilon)$, and $p_1^H(\varepsilon)$ such that $(\varepsilon, x_0^R(\varepsilon), p_0(\varepsilon), p_1^L(\varepsilon), p_1^H(\varepsilon))$ solve the system of equations (and thus constitute an equilibrium) for all ε in a neighborhood of zero. Q.E.D.$_{\text{Proposition 4}}$

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